Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Friday 28 January 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M35409A

1. A particle P moves on the positive x-axis. When the distance of P from the origin O is x metres, the acceleration of P is $(7 - 2x)$ m s^{-2}, measured in the positive x-direction. When $t = 0$, P is at O and is moving in the positive x-direction with speed 6 m s^{-1}. Find the distance of P from O when P first comes to instantaneous rest.

(6)

2.

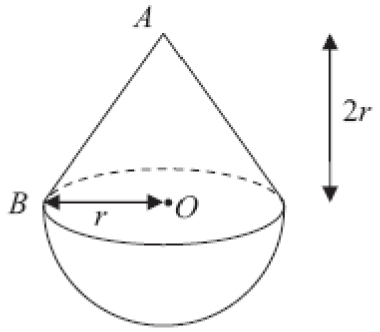

Figure 1

A toy is formed by joining a uniform solid hemisphere, of radius r and mass $4m$, to a uniform right circular solid cone of mass km. The cone has vertex A, base radius r and height $2r$. The plane face of the cone coincides with the plane face of the hemisphere. The centre of the plane face of the hemisphere is O and OB is a radius of its plane face as shown in Figure 1. The centre of mass of the toy is at O.

(a) Find the value of k.

(4)

A metal stud of mass λm is attached to the toy at A. The toy is now suspended by a light string attached to B and hangs freely at rest. The angle between OB and the vertical is 30°.

(b) Find the value of λ.

(4)

3.

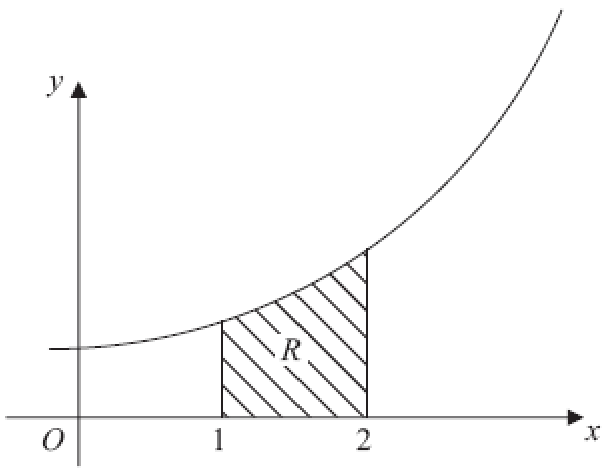

Figure 2

The region R is bounded by the curve with equation $y = e^x$, the line $x = 1$, the line $x = 2$ and the x-axis as shown in Figure 2. A uniform solid S is formed by rotating R through 2π about the x-axis.

(a) Show that the volume of S is $\frac{1}{2}\pi(e^4 - e^2)$.

(4)

(b) Find, to 3 significant figures, the x-coordinate of the centre of mass of S.

(6)

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4. A particle P moves along the x-axis. At time t seconds its displacement, x metres, from the origin O is given by $x = 5\sin(\frac{1}{3}\pi t)$.

(a) Prove that P is moving with simple harmonic motion.

(3)

(b) Find the period and the amplitude of the motion.

(2)

(c) Find the maximum speed of P.

(2)

The points A and B on the positive x-axis are such that $OA = 2$ m and $OB = 3$ m.

(d) Find the time taken by P to travel directly from A to B.

(4)

5.

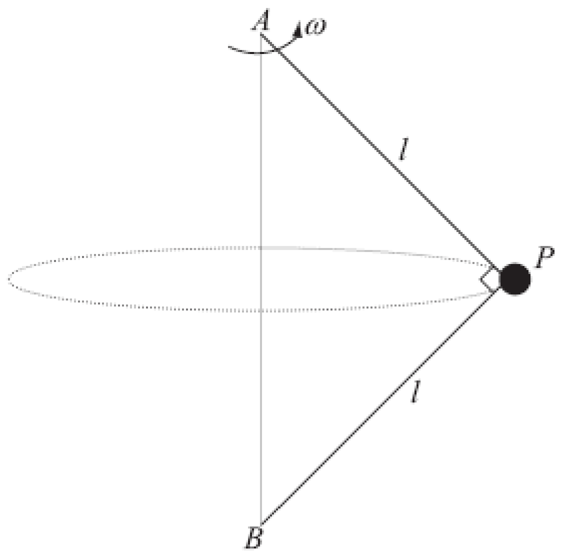


Figure 3

A small ball P of mass m is attached to the ends of two light inextensible strings of length l. The other ends of the strings are attached to fixed points A and B, where A is vertically above B. Both strings are taut and AP is perpendicular to BP as shown in Figure 3. The system rotates about the line AB with constant angular speed ω. The ball moves in a horizontal circle.

(a) Find, in terms of m, g, l and ω, the tension in AP and the tension in BP.

(8)

(b) Show that $\omega^2 > \dfrac{g\sqrt{2}}{l}$.

(2)

6.

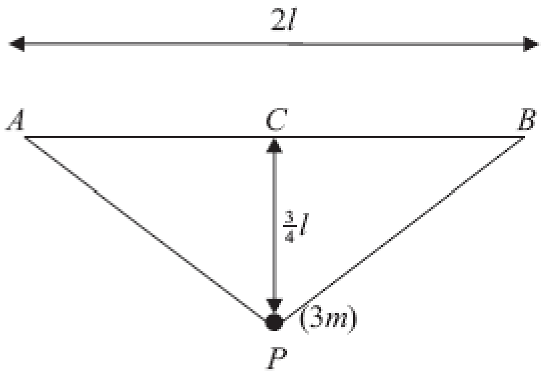


Figure 4

A small ball of mass $3m$ is attached to the ends of two light elastic strings AP and BP, each of natural length l and modulus of elasticity kmg. The ends A and B of the strings are attached to fixed points on the same horizontal level, with $AB = 2l$. The mid-point of AB is C. The ball hangs in equilibrium at a distance $\frac{3}{4}l$ vertically below C as shown in Figure 4.

(*a*) Show that $k = 10$.

(7)

The ball is now pulled vertically downwards until it is at a distance $\frac{12}{5}l$ below C. The ball is released from rest.

(*b*) Find the speed of the ball as it reaches C.

(6)

7.

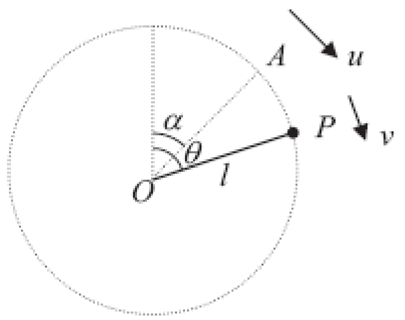


Figure 5

A particle P of mass m is attached to one end of a light rod of length l. The other end of the rod is attached to a fixed point O. The rod can turn freely in a vertical plane about O. The particle is projected with speed u from a point A, where OA makes an angle α with the upward vertical through O and $0 < \alpha < \frac{\pi}{2}$. When OP makes an angle θ with the upward vertical through O the speed of P is v, as shown in Figure 5.

(*a*) Show that $v^2 = u^2 + 2gl(\cos\alpha - \cos\theta)$.

(4)

It is given that $\cos\alpha = \frac{3}{5}$ and that P moves in a complete vertical circle.

(*b*) Show that $u > 2\sqrt{\left(\frac{gl}{5}\right)}$.

(4)

As the rod rotates the least tension in the rod is T and the greatest tension is $5T$.

(*c*) Show that $u^2 = \frac{33}{10}gl$.

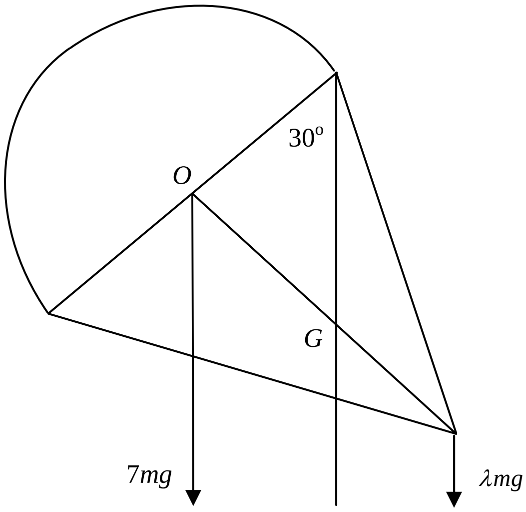
(9)

TOTAL FOR PAPER: 75 MARKS

END

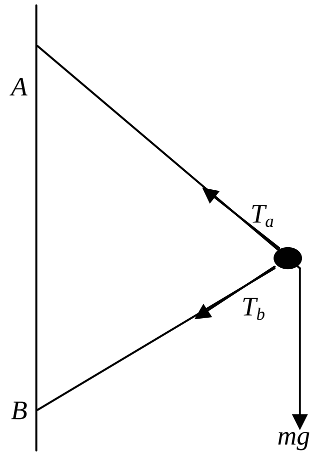
January 2011
Mechanics M3 6679
Mark Scheme

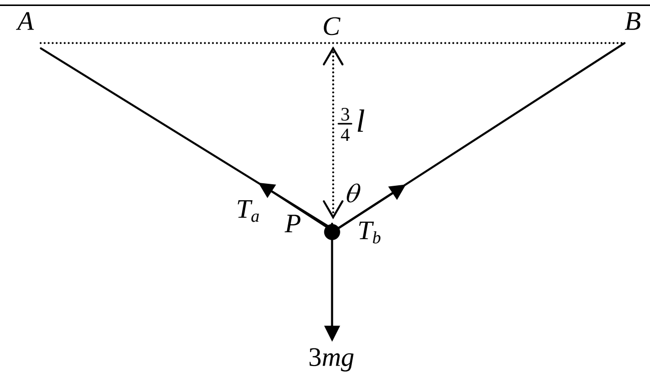
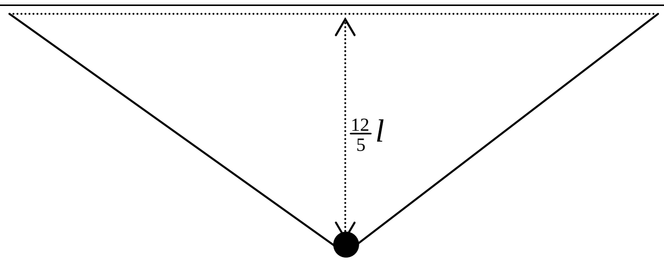
Question Number	Scheme	Marks
1.	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 7 - 2x$	M1
	$\tfrac{1}{2}v^2 = 7x - x^2 \quad (+c)$	M1A1
	$x = 0 \quad v = 6 \quad \Rightarrow \quad c = 18$	A1
	$v = 0 \quad x^2 - 7x - 18 = 0$	
	$(x+2)(x-9) = 0$	M1
	$\therefore x = 9$	A1
		[6]

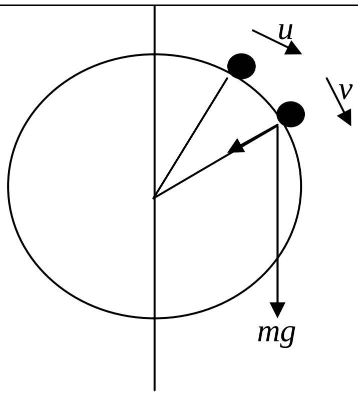
Question Number	Scheme	Marks
2. (a)	Mass ratio $\quad 4m \quad km \quad \Vert \; (4+k)m$ Dist from O $\quad \frac{3}{8}r \quad -\frac{1}{2}r \quad \Vert \; 0$	B1 B1
	Moments about O: $\frac{3}{8}r \times 4m = \frac{1}{2}r \times km$	M1
	$k = 3$	A1
		(4)
(b)		
	$\tan 30 = \frac{OG}{r}$	B1
	$OG = \frac{\lambda}{(7+\lambda)} \times 2r$	M1
	$\frac{1}{\sqrt{3}} = \frac{\lambda}{(7+\lambda)} \times 2r \times \frac{1}{r}$	A1
	$7 + \lambda = 2\sqrt{3}\lambda$ $\lambda = \frac{7}{(2\sqrt{3}-1)}$ (o.e.) or 2.84	A1
		(4) **[8]**

Question Number	Scheme	Marks
3. **(a)**	$\text{Vol } = \pi\int_1^2 y^2\text{d}x = \pi\int_1^2 \text{e}^{2x}\text{d}x$	M1
	$= \frac{1}{2}\pi\left[\text{e}^{2x}\right]_1^2$	M1 A1
	$= \frac{1}{2}\pi\left[\text{e}^4 - \text{e}^2\right]$	A1
		(4)
(b)	$\text{C of M } = \dfrac{\int_1^2 \pi y^2 x\,\text{d}x}{\text{vol}}$	
	$\int_1^2 \text{e}^{2x}x\,\text{d}x = \left[\frac{1}{2}x\text{e}^{2x}\right]_1^2 - \int_1^2 \frac{1}{2}\text{e}^{2x}\,\text{d}x$	M1 A1
	$= \left[\frac{1}{2}x\text{e}^{2x}\right]_1^2 - \left[\frac{1}{4}\text{e}^{2x}\right]_1^2$	M1
	$= \frac{1}{2}\times 2\text{e}^4 - \frac{1}{2}\times 1\text{e}^2 - \left(\frac{1}{4}\text{e}^4 - \frac{1}{4}\text{e}^2\right)$	
	$= \left(\frac{3}{4}\text{e}^4 - \frac{1}{4}\text{e}^2\right)$	A1
	$\text{C of M } = \dfrac{\pi\left(\frac{3}{4}\text{e}^4 - \frac{1}{4}\text{e}^2\right)}{\frac{1}{2}\pi\left(\text{e}^4 - \text{e}^2\right)} = 1.656...$	M1 A1
	$= 1.66$ (3 sf)	
		(6) **[10]**

Question Number	Scheme	Marks
4. **(a)**	$x = 5\sin\left(\frac{\pi t}{3}\right)$ $\dot{x} = 5 \times \frac{\pi}{3}\cos\left(\frac{\pi t}{3}\right)$ $\ddot{x} = -5 \times \left(\frac{\pi}{3}\right)^2 \sin\left(\frac{\pi t}{3}\right)$ $\ddot{x} = -\frac{\pi^2}{9}x \qquad (\therefore \text{S.H.M.})$	 M1A1 A1 (3)
(b)	period = $\frac{2\pi}{\frac{\pi}{3}} = 6$ amplitude = 5	B1 B1 (2)
(c)	$... = 5 \times \frac{\pi}{3}\cos\left(\frac{\pi t}{3}\right) \qquad \text{or} \quad \lvert v_{\max} \rvert = a\omega$ $\max.\ v = \frac{5\pi}{3}$	M1 A1 (2)
(d)	At A $x = 2 \qquad 2 = 5\sin\left(\frac{\pi t}{3}\right)$ $\sin\frac{\pi}{3}t = 0.4$ $t_A = \frac{3}{\pi} \times \sin^{-1} 0.4$ At B $x = 3 \qquad t_B = \frac{3}{\pi} \times \sin^{-1} 0.6$ time $A \to B \quad = \frac{3}{\pi} \times \sin^{-1} 0.6 - \frac{3}{\pi} \times \sin^{-1} 0.4$ $= 0.2215... = 0.22$ s accept awrt 0.22	M1 A1 A1 A1 (4) **[11]**

Question Number	Scheme	Marks
5.		
(a)	$r = \frac{l}{\sqrt{2}}$	B1
	$\text{R}(\uparrow) \quad T_a \cos 45 = T_b \cos 45 + mg$	M1A1
	$T_a - T_b = mg\sqrt{2}$ (1)	
	$\text{R}(\rightarrow) \quad T_a \cos 45 + T_b \cos 45 = mr\omega^2$	M1A1
	$T_a \times \frac{1}{\sqrt{2}} + T_b \times \frac{1}{\sqrt{2}} = m\frac{l}{\sqrt{2}}\omega^2$	
	$T_a + T_b = ml\omega^2$ (2)	
	$T_a - T_b = mg\sqrt{2}$ (1)	
	$2T_a = m\left(l\omega^2 + g\sqrt{2}\right)$	M1
	$T_a = \frac{1}{2}m\left(l\omega^2 + g\sqrt{2}\right)$	A1
	$T_b = ml\omega^2 - T_a$	A1
	$= \frac{1}{2}m\left(l\omega^2 - g\sqrt{2}\right)$	
		(8)
(b)	$T_b > 0 \qquad \frac{1}{2}m\left(l\omega^2 - g\sqrt{2}\right) > 0$	M1
	$\omega^2 > \frac{g\sqrt{2}}{l}$ *	A1
		(2) **[10]**

Question Number	Scheme	Marks
6. **(a)**		
	length AP = length $BP = \frac{5}{4}l$	B1
	$T_a = T_b = \frac{kmg\left(\frac{1}{4}l\right)}{l} = \frac{1}{4}kmg$ $\quad$ (or $T = ...$)	M1A1
	R($\uparrow$) $\quad T_a\cos\theta + T_b\cos\theta = 3mg$ $\quad$ (or $2T\cos\theta = 3mg$)	M1A1
	$\frac{1}{4}kmg\times\frac{3}{5}+\frac{1}{4}kmg\times\frac{3}{5}=3mg$ $\quad$ $\left(\text{or } \frac{1}{2}kmg\times\frac{3}{5}=3mg\right)$	A1
	$\frac{3}{10}kmg = 3mg$	
	$k = 10$ $\quad$ *	A1 (7)
(b)		
	initial extn $= \frac{13}{5}l - l = \frac{8}{5}l$	B1
	E.P.E. lost $= 2\times\frac{\lambda x^2}{2l} = 2\times\frac{10mg}{2l}\left(\frac{8l}{5}\right)^2 = \frac{128mgl}{5}$	M1A1
	P.E. gained $= 3mg\times\frac{12l}{5} = \frac{36mgl}{5}$	
	$\frac{1}{2}\times 3mv^2 + \frac{36mgl}{5} = \frac{128mgl}{5}$	M1A1
	$v^2 = \frac{256gl}{15} - \frac{72gl}{15}$	
	$v = \sqrt{\left(\frac{184}{15}gl\right)}$	A1 (6) **[13]**

Question Number	Scheme	Marks
7.		
(a)	$mgl\left(\cos\alpha-\cos\theta\right)=\frac{1}{2}mv^2-\frac{1}{2}mu^2$	M1A1=A1
	$v^2=u^2+2gl\left(\cos\alpha-\cos\theta\right)$ *	A1
		(4)
(b)	$\cos\alpha=\frac{3}{5}$ $\quad v^2=2gl\left(\frac{3}{5}-\cos\theta\right)+u^2$	
	At top $\theta=360°$ $\quad v^2=2gl\left(\frac{3}{5}-1\right)+u^2$	M1A1
	$v^2>0 \quad -2gl\times\frac{2}{5}+u^2>0$	M1
	$u^2>\frac{4gl}{5}$	
	$u>2\sqrt{\frac{gl}{5}}$ *	A1
		(4)

Question Number	Scheme	Marks
(c)	Equation of motion along radius at lowest point:	
	$T_1 - mg = \frac{mv^2}{l}$	M1A1
	$\theta = 180 \qquad v^2 = 2gl\left(\frac{3}{5}+1\right)+u^2$	M1
	$v^2 = \frac{16gl}{5}+u^2$	
	$T_1 = \frac{m}{l}\left(\frac{16gl}{5}+u^2\right)+mg$	
	$= \frac{21mg}{5}+\frac{mu^2}{l}$	A1
	At highest point:	
	$T_2 + mg = \frac{mv^2}{l}$	M1
	$\theta = 360 \qquad T_2 = 2mg\left(-\frac{2}{5}\right)+\frac{mu^2}{l}-mg$	M1
	$T_2 = \frac{mu^2}{l}-\frac{9mg}{5}$	A1
	$T_1 = 5T_2$	
	$\frac{21mg}{5}+\frac{mu^2}{l} = 5\left(\frac{mu^2}{l}-\frac{9mg}{5}\right)$	M1
	$\frac{66mg}{5} = \frac{4mu^2}{l}$	
	$u^2 = \frac{33gl}{10} \qquad *$	A1
		(9)
		[17]